COMMON PRE-BOARD EXAMINATION 2022-23

CLASS: XII

SUBJECT: MATHEMATICS (041)

Date:

<u>Time Allowed: 3 Hours</u>   Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions. Each question carries 1 mark)

1. For any square matrix A, AA^T is a

   (a) unit matrix   (b) symmetric matrix

   (c) skew-symmetric matrix   (d) diagonal matrix

2. If B is a non-singular matrix and A is a square matrix, then $\det(B^{-1}AB)$ is equal to

   (a) $\det(A^{-1})$ (b) $\det(B^{-1})$

   (c) $\det(A)$ (d) $\det(B)$

3. The angle between a line whose direction ratios are in the ratio 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12) is

   (a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(-\frac{2}{3}\right)$

   (c) $\tan^{-1}\left(\frac{2}{3}\right)$ (d) None of these

4. The differential equation satisfied by the function

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots + \infty}}} \text{ is}$$

(a) $(2y - 1)\frac{dy}{dx} - \sin x = 0$

(b) $(2y - 1)\cos x + \frac{dy}{dx} = 0$

(c) $(2y - 1)\cos x - \frac{dy}{dx} = 0$

(d) $(2y - 1)\frac{dy}{dx} - \cos x = 0$

5. If $\int \frac{e^x(1+\sin x)\,dx}{1+\cos x} = e^x f(x) + C$ is equal to
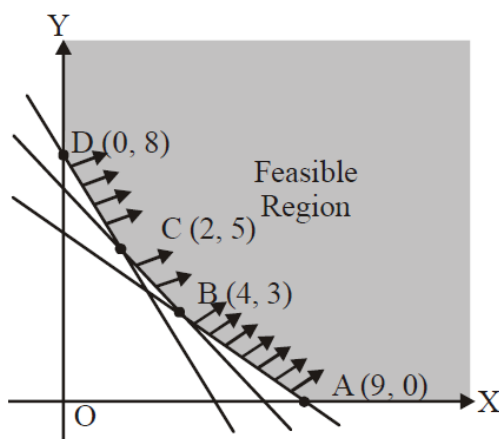
(a) $\sin\frac{x}{2}$

(b) $\cos\frac{x}{2}$

(c) $\tan\frac{x}{2}$

(d) $\log\frac{x}{2}$

6. Integrating factor of the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$

(a) $\cos x$

(b) $\sec x$

(c) $e^{\cos x}$

(d) $e^{\sec x}$

7. The feasible region for an LPP is shown shaded in the figure.

Let Z = 4 x + 3y be the objective function. Minimum of Z occurs at

(a) (0, 8)   (b) (2, 5)   (c) (4, 3)   (d) (9, 0)

8. If $|\vec{a}| = 5$, $|\vec{b}| = 4$, $|\vec{c}| = 3$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of

$|\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}|$ is equal to

(a) 25   (b) 50   (c) -25 (d) -50

9. Value of $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx$ is

   (a) 2   (b) 3   (c) 4   (d) 5

10. If matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{k}(\text{adj}A)$, then value of k is

   (a) 7   (b) −7   (c) −11   (d) 15

11. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let Z $=$ px $+$ qy, where p, q > 0. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is

   a) p = 2 q   (b) $p = \frac{q}{2}$ (c) p = 3 q   (d) p = q

12. For any 2×2 matrix A, if $A(\text{adj}A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ is equal to

   (a) 0   (b) 10   (c) 20   (d) 100

13. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x equals

   (a) 2   (b) $-\frac{1}{2}$ (c) 1   (d) $\frac{1}{2}$

14. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then P(B) is

   (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

15. If p and q are the order and degree of the differential equation

$$y\frac{dy}{dx} + x^3\frac{d^2y}{dx^2} + xy = \cos x, \text{ then}$$

   (a) $p < q$ (b) $p = q$ (c) $p > q$ (d) None of these

16. For $y = \cos kx$ to be a solution of differential equation $\frac{d^2y}{dx^2} + 4y = 0$, the value of k is

   (a) 2   (b) 4   (c) 6   (d) 8

17. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

   (a) $\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ (c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

18. The vector equation of the line through the point (5, 2, – 4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$ is

   (a) $(5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$

   (b) $(5 - 3\lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (4 + 8\lambda)\hat{k}$

   (c) $(5 + 3\lambda)\hat{i} - (2 + 2\lambda)\hat{j} - (-4 - 8\lambda)\hat{k}$

   (d) $(3 + 5\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-8 - 4\lambda)\hat{k}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A)**: The value of $\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is 1.

   Reason (R): $\tan^{-1}(-x) = \tan x$ and $\cos^{-1}(-x) = \cos^{-1} x$

20. **Assertion (A)**: The lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

   Reason (R): The lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are perpendicular to each other if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. Find the value of x if $2\tan^{-1}(\cos x) = \tan^{-1}(2\cosec x)$.

OR

Check whether the function $f: R \to R$ given by $f(x) = x^3 - 1$ is bijective or not.

22. A stone is dropped into a quiet lake and waves moves in circles at the speed of 5 cm/s. If at an instant, the radius of the circular wave is 8 cm, then find the rate at which enclosed area is increasing.

23. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

OR

Find the angle between the pair of lines given by

$\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = (5\hat{\imath} - 2\hat{\jmath}) + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$

24. If $e^y(x + 1) = 1$, show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$.

25. Let $\vec{a} = 3\hat{\imath} - 5\hat{\jmath}$ and $\vec{b} = 6\hat{\imath} + 3\hat{\jmath}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a} \times \vec{b}$. Find the value of $|\vec{a}| : |\vec{b}| : |\vec{c}|$.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find $\int \dfrac{1}{\sqrt{7 - 6x - x^2}}\, dx$

27. Given three identical boxes I, II and III, each containing two coins. Inbox I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

OR

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{other wise} \end{cases}$$

(a) Find the value of k.

(b) What is the probability that you study at least two hours?

(c) What is the probability that you study at most two hours?

28. Evaluate: $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} \, dx$

OR

Evaluate: $\int_{-1}^{\frac{3}{2}} |x \sin(\pi x)| \, dx$

29. Find a particular solution of the differential equation

$\frac{dy}{dx} + 2y\tan x = \sin x; y = 0$ when $x = \frac{\pi}{3}$

OR

Find a particular solution of the differential equation $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1$ given that $y = 0$ when $x = 0$.

30. Solve the following Linear Programming Problem graphically:

Maximize: $Z = 20x + 10y$

Subject to $x + 2y \le 8, 3x + y \le 9, \ x \ge 2, \ x \ge 0, y \ge 0$

31. Find $\int \frac{x^2+x+1}{(x+2)(x^2+1)} \, dx$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Make a rough sketch of the region $\{(x, y): x^2 + y^2 \leq 4, x + y \geq 2\}$ and find the area of the region using integration.

33. Show that the relation R defined on the set $N \times N$ by

$(a, b)R(c, d)$ iff $ad(b + c) = bc(a + d)$ is an equivalence relation.

OR

Show that the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Also find the set of all elements related to 1.

34. Find the vector and Cartesian equation of a line passing through $(1, 2, -4)$ and perpendicular to the two lines:

$\dfrac{x-8}{3} = \dfrac{y+19}{-16} = \dfrac{z-10}{7}$ and $\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$

OR

Find the coordinates of the foot of the perpendicular and length of perpendicular drawn from the point $P(5,4,2)$ to the line $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$.

35. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations: $2x + y - 3z = 13$; $3x + 2y + z = 4$; $x + 2y - z = 8$

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub –parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:** Let $P(x) = -5x^2 + 125x + 37500$ is the total profit function of a bike manufacture company, where x is the production of the company.

Based on the above information, answer the following questions.

(i) What will be production of the company when the profit is ₹38250?

(ii) When the production is 2 unit, what will be profit of the company?

(iii) Find the maximum profit of the company.

OR

Find the intervals in which the profit is strictly increasing and decreasing.

37. **Case-Study 2:**

A poster is to be formed for a company advertisement. The top and bottom margins of poster should be 9cm and the side margins should be 6cm. Also the area for printing the advertisement should be 864cm^2.

Based on the above information, answer the following questions.

(i) If a cm be the width and b cm be the height of poster, then express the area of poster in terms of a and b.

(ii) If a cm be the width and b cm be the height of poster, then find the relation between a and b.

(iii) Find the height of the poster at which the area of the poster is minimum.

OR

Find width of the poster at which the area of the poster is minimum.

38. **Case-Study 2:** A factory has three machines A, B and C to manufacture bolts. Machine A manufacture 30%, Machine B manufacture 20% and Machine C manufacture 50% of the bolts respectively. Out of their respective outputs 5%, 2% and 4% are defective. A bolt is drawn at random from total production and it is found to be defective.

Based on the above information, answer the following questions.

(i) Find the probability that defective bolt drawn is manufactured by Machine A.

(ii) Find the probability that defective bolt drawn is not manufactured by Machine B.

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